MATH 102 (CALCULUS II)

SUPPLEMENTARY EXERCISES

(CHAPTER 6)

Q. 1. Show that the inverse of the following functions exist and also find the inverse function of each of the followings:

(i) $f(x) = e^{\sqrt{x}} + 1$ (June 1996)

(ii) $f(x) = e^{x^2} - 1 \ (x < 0)$ (February 1997)

(iii) $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$ (June 1990)

(iv) $f(x) = \dfrac{3x+2}{2x-3}$ (Feb. 1987)

(v) If $f(x) = \ln\left(\dfrac{e^x}{1+x^2}\right) + x + 1$, then find the equation of tangent line to the graph f^{-1} at the point $P(1,0)$. (1993)

Q.2. Show that the following identities are true:

(i) $\tanh\left(\dfrac{\ln x}{2}\right) = \dfrac{x-1}{x+1}$ (June 1993)

(ii) $\left[\cosh\left(\dfrac{x}{4}\right) + \sinh\left(\dfrac{x}{4}\right)\right]^4 = \cosh x + \sinh x$ (1996)

(iii) $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$

Q.3. Find $\dfrac{dy}{dx}$ if

(i) $y = \left(\sqrt{x}\right)^{\tan^{-1} x} + x^{\ln x}$ (1995)

(ii) $\sqrt{y} = \dfrac{x^x \sin^{-1} x}{(3-2x)\sqrt{x}}$ (1993)

(iii) $ye^x + e^y \ln x + x + 3 = 0$ (1994)

Q.4. Evaluate the following integrals:

(i) $\displaystyle\int \frac{dx}{e^x + 1}$ (1989)

(ii) $\displaystyle\int \frac{\log_5\left(5^x \sqrt{x^3}\right)dx}{x}$ (1993)

(iii) $\displaystyle\int \frac{e^{2x}dx}{\left(1+2e^{2x}\right)^2}$ (1987)

(iv) $\displaystyle\int \frac{\left(2^{x+1} - 2^{3x}\right)^2}{2^{5x}}dx$ (1992)

Q.5. Evaluate the following limits:

(I) $\displaystyle\lim_{x\to\infty}\left(\frac{xe^x}{e^x - 1}\right)$ (1995)

(ii) $\displaystyle\lim_{x\to\infty} (1+3x)^{\frac{1}{\ln x}}$ (1992)

(iii) $\displaystyle\lim_{x\to 0}\left(\frac{\ln(1+2x) - 2x}{x^2}\right)$ (1986)